Josephson Tunnel Junctions with Nonlinear Damping
for RSFQ-Qubit Circuit Applications

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We demonstrate that shunting of Superconductor-Insulator-Superconductor (S-I-S) Josephson junctions by Superconductor-Insulator-Normal metal (S-I-N) structures having pronounced nonlinear I-V characteristics can remarkably modify the Josephson dynamics. In the regime of Josephson generation the phase behaves as an overdamped coordinate, while in the superconducting state the damping and current noise are strikingly small, that is vitally important for application of such junctions for readout and control of Josephson qubits. Superconducting Nb/AlO$_x$/Nb junction shunted by Nb/AlO$_x$/AuPd junction of S-I-N type was fabricated and, in agreement with our model, exhibited non-hysteretic I-V characteristics at temperatures down to at least 1.4 K.

The *overdamped* Josephson junctions are the key elements of the SQUIDs and Rapid Single-Flux-Quantum (RSFQ) logic circuits [1, 2]. Due to non-hysteretic (single-valued) I-V characteristics, they allow the convenient readout of critical currents of interferometers and the generation and processing of Single Flux Quantum (SFQ) voltage pulses associated with 2π slips of the Josephson phase. Recently, RSFQ electronics have been considered as possible complementary digital electronics for serving the Josephson qubits. Specifically, these generic superconducting circuits should allow the efficient control, readout and processing of information in terms of the SFQ pulses (see, for example, Refs. [3, 4]). Among the challenging problems of matching RSFQ and qubit circuits, the principal one seems to be the back action of RSFQ circuits on qubit [3, 5]. This back action is due to current noise which stems from unfavorably large damping in the RSFQ-circuit junctions. Moreover, this noise is essential even in the quiescent (zero-voltage) state of the junctions. By means of direct inclusion (or inductive coupling) of an RSFQ-circuit input in (to) a qubit based on underdamped Josephson junctions, the current (or flux) noise may cause dramatic decoherence of the qubit: pure dephasing due to low-frequency components of the noise and both dephasing and relaxation due to the frequency components close to the transition frequency of the qubit ν_{01} [6]. (Typically, $\nu_{01} = $ 10-30 GHz $\ll \Delta/h$, Δ being the superconductor energy gap.) Then quantum manipulation of the qubit is impossible.

At an operating temperature $T \lesssim 0.5 T_c$ (T_c being the critical temperature of the superconductor) the most technological Josephson junctions of the Superconductor-Insulator-Superconductor (S-I-S) type of low and moderate critical current density j_c are *underdamped* [7]. In these junctions, the transient dynamics of the Josephson phase ϕ prevents their immediate use for generating SFQ pulses. When the bias current I rises above the critical value I_c, the junction switches from the superconducting to the resistive state, while reset (return to the superconducting state) occurs at the much smaller current $I_R \ll I_c$. Overdamped behavior is usually achieved by

shunting the junctions with a normal resistance of sufficiently small R. In this case, the dynamics is described by the Resistively Shunted Junction (RSJ) model [8, 9] and the condition of sufficient damping is formulated as $\beta_c \lesssim 2$, where the Stewart-McCumber parameter is given by

$$\beta_c = (2\pi/\Phi_0)I_c R^2 C, \qquad (1)$$

$\Phi_0 = h/2e$, flux quantum and C, junction capacitance. For such β_c, the hysteresis in autonomous junctions is small, $I_R/I_c \gtrsim 80\%$, which ensures correct operation of an RSFQ circuit. The real part of the junction zero-voltage admittance is, however, large, $\mathrm{Re}Y(\omega) = G \equiv R^{-1}$. The corresponding spectral density of current fluctuations,

$$S_I(\omega) = (\hbar\omega G/\pi) \coth(\hbar\omega/2k_B T), \qquad (2)$$

is also large at the qubit frequencies, i.e., $\omega/2\pi \lesssim \nu_{01}$.

One of the possible ways to suppress the current fluctuations consists in using a very transparent tunnel barrier having a specific tunnel resistance $\rho \sim 1\,\Omega \cdot \mu\mathrm{m}^2$ yielding critical current density $j_c = 10^5 - 10^6\,\mathrm{A/cm}^2$ [10, 11, 12]. In the dynamic state ($V \neq 0$), the effective damping dramatically increases and the return current I_R can approach I_c [13, 14, 15]. At $T \to 0$ and $\omega < 2\Delta/\hbar$, the zero-bias admittance $\mathrm{Re}Y(\omega) = \sigma_{\mathrm{qp}}(\omega) + \sigma_{\mathrm{p\text{-}qp}}(\omega)\cos\phi$ (where $\sigma_{\mathrm{qp}}(eV/\hbar)$ is quasiparticle and $\sigma_{\mathrm{p\text{-}qp}}(eV/\hbar)$ is pair-quasiparticle-interference conductances [16]), is remarkably small (for the BCS density of states, $\mathrm{Re}Y \propto \exp(-\Delta/k_B T)$ [17, 18]). So one could expect low current noise in this frequency range. Unfortunately, the barriers of these junctions often have pin-hole defects leading to large σ_{qp} at small V [12]. Therefore, these junctions can hardly be implemented in multi-junction circuits and guarantee small noise.

In this Letter we frame a concept of *nonlinear* shunts which can be realized by means of Superconductor-Insulator-Normal metal (S-I-N) junctions. It is well known that the I-V curve of an S-I-N junction is strongly
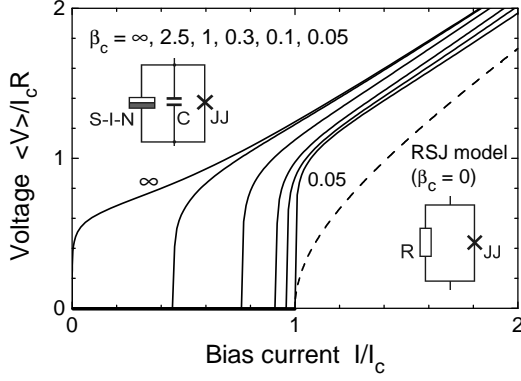
FIG. 1: *I-V* curves of S-I-N shunted Josephson junction calculated for different values of parameter β_c with the bias current sweep starting from the large values (solid curves). The values of the other parameters are: $\delta \equiv \Delta/k_B T = 10$ and $\alpha \equiv eI_c R/\Delta = \pi/2$. The dashed line is the hyperbolic-shaped *I-V* curve given by the RSJ model with zero capacitance [8, 9].

FIG. 2: Return current versus parameter β_c calculated in our model for different values of δ and fixed $\alpha = \pi/2$ (solid lines) and RSJ model (dashed line) [8].

non-linear. For the BCS superconductor, it is given by

$$I_{\mathrm{SIN}}(V) = \frac{1}{eR} \int_\Delta^\infty dE \frac{[f(E - eV) - f(E + eV)]E}{(E^2 - \Delta^2)^{1/2}}, \quad (3)$$

R being the junction resistance at large voltage ($eV \gg \Delta$) and $f(E) = 1/[1 + \exp(E/k_B T)]$, the Fermi function. At low temperature, $\delta = \Delta/k_B T \gg 1$, the small-bias current $I_{\mathrm{SIN}}(V \ll \Delta/e)$ and the zero-bias conductance G_0 of the junction are small, $G_0 R = (2\pi\delta)^{1/2} \exp(-\delta) \ll 1$ [19]. Therefore, one can expect only small current noise to be generated by this element at $\omega \ll \Delta/\hbar$ (see Eq. (2) in which G should be substituted for $eI_{\mathrm{SIN}}(\hbar\omega/e)/\hbar\omega$ [20]). The only question which arises is whether it is possible to ensure sufficient damping in a Josephson junction by shunting it with an S-I-N junction.

The answer is apparently positive, first of all, because the dynamics of S-I-S junctions with very small quasiparticle leakage current can theoretically be made overdamped by increasing j_c. Quantitatively, the condition can be formulated in terms of $\beta_c \lesssim 0.1$, assuming that R in Eq. (1) is the tunneling resistance of the S-I-S junction at $V \gg 2\Delta/e$ (see calculations made within the framework of the microscopic model of Josephson tunneling [17, 18] in Ref. [15]). The simpler model of Prober et al. [21], approximating the quasiparticle curve by three straight line segments, also gives the values of β_c sufficient for suppressing the hysteresis in the range of $0.1-1$.

We carried out simple simulations of a Josephson junction shunted by an S-I-N junction. In this model we assumed that the total current I from a source branches into the capacitive, dissipative and Josephson components:

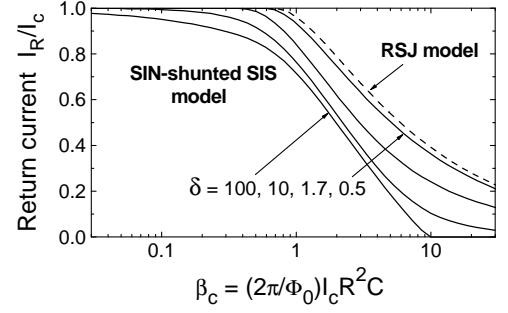$$C\frac{dV}{dt} + I_{\mathrm{SIN}}(V) + I_c \sin\phi = I, \quad (4)$$

where C is the total capacitance of both junctions. Current $I_{\mathrm{SIN}}(V)$ is given by the BCS dependence Eq. (3) with the instant voltage V as an argument. The Josephson current amplitude I_c was assumed to be frequency-independent and the quasiparticle and pair-quasiparticle interference components of current through the S-I-S junction were omitted. These assumptions are reasonable because in the interesting voltage range (up to approximately the characteristic value $V_c \equiv I_c R$), the damping should be produced by the S-I-N junction. Finally, we omitted the ac terms in the current through the S-I-N junction, related to the imaginary part of its impedance and leading to an equation of motion of the integral type. The estimation had, however, shown that taking these terms in account did not lead to an essential correction of the dc *I-V* curves, but significantly complicated the calculations.

We numerically solved Eq. (4) for different values of parameter δ. We found out that in the most interesting case of pronounced nonlinearity, i.e. large δ ($\gtrsim 10$), sufficient damping can be achieved by reasonable reduction of β_c only if the parameter $\alpha \equiv eV_c/\Delta \gtrsim 1$. Figure 1 shows the effect of β_c on the *I-V* curves for fixed $\delta = 10$ (yielding the ratio of the zero-bias and large-bias conductances $G_0 R \approx 3 \times 10^{-4}$) and $\alpha = \pi/2$. The leftmost curve which corresponds to $\beta_c = \infty$ is merely the *I-V* curve of the S-I-N junction for the given δ. At $\beta_c < 1$, hysteresis is sufficiently suppressed. The shape of these curves has a much steeper voltage rise at $I = I_c$ than that given by the RSJ model (dashed line). This is the result of deficient low-frequency damping which the system compensates by large damping at high frequencies, corresponding to large mean voltage, $\langle V \rangle \geq \Delta/e$. The return current as function of β_c (see Fig. 2) generally mimics the dependencies given by the RSJ model, especially for small δ (cf. dashed line), as well as the piecewise-resistor model [21] and the microscopic model [15] (both not shown).

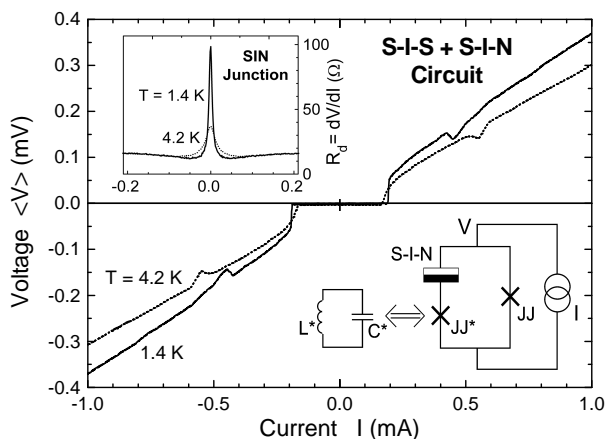To implement the idea, we manufactured S-I-S junctions from Nb/AlO$_x$/Nb trilayer with a nominal critical

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FIG. 3: Non-hysteretic I-V curves of S-I-N-shunted SIS junction measured at $4.2\,\mathrm{K}$ (dotted line) and $1.4\,\mathrm{K}$ (solid line). Upper inset shows the dV/dI-I curves of a stand-alone S-I-N junction manufactured from the same sandwich. Lower inset shows the electric diagram of the circuit including an additional S-I-S junction JJ* which was formed beneath the S-I-N structure because of merely technological simplicity.

current density of $j_c = 500\,\mathrm{A/cm^2}$ (see Ref. [22] for details of the process). The upper Nb layer was coated with a thin Al layer which was then oxidized forming the insulator for the S-I-N sandwich. For the normal metal counter electrode we used a relatively thick (100 nm) layer of AuPd alloy (normally used for thin-film on-chip resistors). The specific tunnel resistance of the S-I-N structure was $\rho \approx 135\,\Omega \cdot \mu\mathrm{m}^2$. The areas of S-I-S and S-I-N junctions were $24\,\mu\mathrm{m}^2$ and $450\,\mu\mathrm{m}^2$, respectively. The characteristics of the S-I-N sandwich are shown in upper inset of Fig. 3. The product $G_0 R$ was found to be significantly smaller than expected which is possibly due to imperfection in this rather transparent barrier. Nevertheless, these values (≈ 2.3 at $4.2\,\mathrm{K}$ increasing up to 6.2 at $1.4\,\mathrm{K}$ and yielding a fit value $\delta = 3.3$) still allowed rather nonlinear shunts to be realized. The resulting non-hysteretic I-V curves unambigously indicate the overdamped regime of operation. The estimated values of β_c are about 1. The curve measured at $4.2\,\mathrm{K}$ is close to that given by the RSJ model, while the shape of the curve at $1.4\,\mathrm{K}$ is similar to that found in our model.

The I-V curves in Fig. 3 also show peculiarities due to the presence of an additional S-I-S junction formed in the present technology directly beneath the S-I-N sandwich and connected in series to it. Since its critical current was sufficiently large, $I_c^* \approx 2.7\,\mathrm{mA} \gg I_c$, it remained in the superconducting state even at $I_c < I \leq I_c^*$. Its equivalent circuit is displayed as parallel connection of Josephson inductance $L^* = \Phi_0/2\pi I_c^*$ and capacitance C^* (see lower inset in Fig. 3) yielding a plasma frequency $\nu_p = 1/2\pi(L^*C^*)^{1/2} \approx 70\,\mathrm{GHz}$. Such a shunting path almost behaved as a sole S-I-N junction when the Joseph-

son oscillation frequency $2e\langle V \rangle/h$ was not close to ν_p. At $\langle V \rangle \approx h\nu_p/2e$ the impedance of the shunting path resonantly increased and this enhanced (negative) contribution to the average voltage due to self-detecting of Josephson oscillations. As a result, the negative peaks with tips positioned at $\langle V \rangle \approx \pm 140\,\mu\mathrm{V}$ had emerged.

In conclusion, we have shown by modelling and preliminary experiment that external shunting of S-I-S Josephson junctions by S-I-N junctions can ensure both sufficiently high dynamic damping and low zero-voltage damping. For operation at very low temperature ($\delta \gg 1$), the conditions $\beta_c \lesssim 1$ and $\alpha \gtrsim 1$ lead to a limitation of the specific resistance of S-I-N structure, viz. $\rho \lesssim \hbar/2c\Delta$, c being the specific capacitance of the barrier. For example, in the S-I-N structures based on superconducting aluminum ($\Delta_{\mathrm{Al}} \approx 200\,\mu\mathrm{eV}$, $c \approx 50\,\mathrm{fF}/\mu\mathrm{m}^2$) and operating at qubit (mK) temperatures, ρ should be $\lesssim 30\,\Omega \cdot \mu\mathrm{m}^2$. The manufacture of such sandwiches ensuring negligible zero-bias conductance required for dramatic suppression of back action on the qubit is quite feasible.

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